UNIFICATION OF CERTAIN GENERALIZED POLYNOMIAL SET $B_n(x_1, x_2, x_3)$ ASSOCIATED WITH LAURICELLA FUNCTIONS

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ABSTRACT

In the present paper a generalized Hypergeometric Polynomial Set $B_n(x_1, x_2, x_3)$ has been defined by means of a generating function which contain product of two gneralized Hypergeometric function and Lauricella functions in the notation of Burchnall and Choundy [4]. This polynomial set covers as many as thirty eight orthogonal and non-orthogonal polynomial have been obtained with three special cases. a number of known orthogonal and non-orthogonal polynomials have been deduced from three special cases.

Keywords : Hypergeometric Polynomial, Lauricella functions Orthogonal Polynomial.

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1. INTRODUCTION

We define the generalized polynomial set $B_n(x_1, x_2, x_3)$ by means of generating relation.

$$\left(\nu_\lambda + \mu_1 x_1^{(c-d)} t^{r_1}\right)^{-\sigma_1} \times \left[\begin{matrix} (C_p);(E_g);(\alpha_u)(\alpha_m^1) \\ \mu x_1^r t, \mu_2 x_2^{r_1} t^{r_1}, \mu_3 x_3^{r_1} t^{r_2} \\ (D_q);(F_h);(\beta_v);(\beta_k^1) \end{matrix}\right]$$

$$= \sum_{n=0}^{\infty} B^{\mu;\mu_1;\mu_2;\mu_3;\sigma_1;a;c;d;(C_p);(E_g);(\alpha_u)(\alpha_m^1);}_{n,r;r_1;r_2;r_3;(D_q);(F_h);(\beta_v);(\beta_k^1)} \left(x_1, x_2, x_3\right) t^n \qquad \ldots (1.1)$$

where μ, μ_1, μ_2, μ_3, σ_1, a, c, d, are real numbers and r and r_1 are non-negative integer and r_2, r_3 are natural numbers. The left hand side of (1.1) contains the product of generalized hypergeometric function and Lauricella function in the notation of Burchanall and Chaundy. The generalized polynomial set contains number of parameters.

For Simplicity we shall denote

$$B^{\mu;\mu_1;\mu_2;\mu_3;\sigma_1;a;c;d;(C_p);(E_g);(\alpha_u)(\alpha_m^1);}_{n,r;r_1;r_2;r_3;(D_q);(F_h);(\beta_v);(\beta_k^1)} \left(x_1, x_2, x_3\right)$$

by $B_n(x_1, x_2, x_3)$

where n denotes the order of the polynomial set.

NOTATIONS

I. $(m) = 1, 2, 3, \ldots\ldots, m.$
II. $(A_p) = A_1, A_2, A_3, \ldots\ldots A_p.$
III. $[(A_p)] = A_1, A_2, A_3, \ldots\ldots A_p.$
IV. $[(A_p)]_n = (A_1)_n, (A_2)_n, (A_3)_n \ldots\ldots (A_p)_n.$
V. $\Delta(a,b) = \frac{b}{a}, \frac{b+1}{a}, \ldots\ldots \frac{b+a-1}{a}.$

2. THEOREM

If the generlized polynomial set is defined by (1.1), them

$$B_n(x_1,x_2,x_3) = \nu_n^{\sigma_1} \sum_{s_1=0}^{\left[\frac{n}{r_1}\right]} \sum_{s_2=0}^{\left[\frac{n-r_1s_1}{r_2}\right]} \sum_{s_3=0}^{\left[\frac{n-r_1s_1-r_2s_2}{r_3}\right]}$$

$$\times \frac{\left[(C_p)\right]_{n-r_1s_1-(r_2-1)s_2-(r_2-1)s_3} \left[(E_s)\right]_{n-r_1s_1-r_2s_2-r_3s_3}}{\left[(D_q)\right]_{n-r_1s_1-(r_2-1)s_2-(r_2-1)s_3} \left[(F_h)\right]_{n-r_1s_1-r_2s_2-r_3s_3}}$$

$$\times \frac{\left[(\alpha_u)\right]_{s_2} \left[(\alpha_m^1)\right]_{s_3} \left[(\sigma_1)\right]_{s_1} \left(\mu x_1^r\right)^{n-r_1s_1-r_2s_2-r_3s_3} \mu_1^{s_1} \left(\mu_2 x_2^{r_2}\right)^{s_2} \left(\mu_3 x_3^{r_3}\right)^{s_3}}{\left[(\beta_v)\right]_{s_2} \left[(\beta_k^1)\right]_{s_3} (n-r_1s_1-r_2s_2-r_3s_3)!\ \nu_n^{s_1}\ s_2!\ s_3!} \quad \ldots (2.1)$$

Proof : We have from (1.1)

$$\sum_{n=0}^{\infty} B_n(x_1,x_2,x_3)\, t^n \sum_{n=0}^{\infty} \sum_{s_1,s_2,s_3=0}^{\infty} \frac{\nu_n^{-\sigma_1} (\sigma_1)_{s_1} \left(\mu_1 t^{r_1}\right)^{s_1} x_1^{(c-d)r_1s_1}}{s_1!\ \nu_n^{s_1}}$$

$$\times \frac{\left[(C_p)\right]_{n+s_2+s_3} \left[(E_g)\right]_n \left[(\alpha_u)\right]_{s_2} \left[(\alpha_m^1)\right]_{s_3} \left(\mu x_1^r t\right)^n \left(\mu_2 x_2^{r_2} t^{r_2}\right)^{s_2} \left(\mu_3 x_3^{r_3} t^{r_3}\right)^{s_3}}{\left[(D_q)\right]_{n+s_2+s_3} \left[(F_h)\right]_n \left[(\beta_v)\right]_{s_2} \left[(\beta_k^1)\right]_{s_3}\ n!\ s_2!\ s_3!}$$

$$= \nu_s^{-\sigma_1} \sum_{n=0}^{\infty} \sum_{s_1,s_2,s_3=0}^{\infty} \frac{\left[(C_p)\right]_{n+s_2+s_3} \left[(E_s)\right]_n \left[(\alpha_u)\right]_{s_2} \left[(\alpha_m^1)\right]_{s_3}}{\left[(D_q)\right]_{n+s_2+s_3} \left[(F_h)\right]_n \left[(\beta_v)\right]_{s_2} \left[(\beta_k^1)\right]_{s_3}}$$

$$\times \frac{(\sigma_1)_{s_1} \mu_1^{s_1} x_1^{(c-d)r_1s_1} \left(\mu x_1^r\right)^n \left(\mu_2 x_2^{r_2}\right)^{s_2} \left(\mu_3 x_3^{r_3}\right)^{s_3}}{s_1!\ \nu_n^{s_1}\ n!\ s_2!\ s_3!}$$

$$\times\, t^{n+r_1s_1+r_2s_2+r_3s_3} \quad \ldots (2.2)$$

$$= \nu_a^{-\sigma_1} \sum_{n=0}^{\infty} \sum_{s_1=0}^{\left[\frac{n}{r_1}\right]} \sum_{s_2=0}^{\left[\frac{n-r_1s_1}{r_2}\right]} \sum_{s_3=0}^{\left[\frac{n-r_1s_1-r_2s_2}{r_3}\right]} \frac{\left[(C_p)\right]_{n-r_1s_1-(r_2-1)s_2-(r_2-1)s_3}}{\left[(D_q)\right]_{n-r_1s_1-(r_2-1)s_2-(r_2-1)s_3}}$$

$$\times \frac{\left[(E_g)\right]_{n-r_1s_1-r_2s_2-r_3s_3} \left[(\alpha_u)\right]_{s_2} \left[(\alpha_m^1)\right]_{s_3} (\sigma_1)_{s_1} \mu_1^{s_2}}{\left[(F_h)\right]_{n-r_1s_1-r_2s_2-r_3s_3} \left[(\beta_v)\right]_{s_2} \left[(\beta_k^1)\right]_{s_3} \; s_1! \; \nu_n^{s_1}}$$

$$\times \frac{x_1^{(c-d)r_1s_1} \left(\mu x_1^r\right)^{n-r_1s_1-r_2s_2-r_3s_3}}{(n-r_1s_1-r_2s_2-r_3s_3)!} \frac{\left(\mu_2 x_2^{r_2}\right)^{s_2}}{s_2!} \frac{\left(\mu_3 x_3^{r_3}\right)^{s_3}}{s_3!} t^n \qquad \ldots (2.3)$$

Equating the Co-efficient of tn from both sides of (2.3), we get (2.1)
After little simplification (2.1) can be written as

$$B_n(x_1, x_2, x_3) = R\nu_a^{-\sigma_1} \frac{\left[1-(D_q)-n\right]_{r_1s_1+(r_2-1)s_2+(r_3-1)s_3}}{\left[1-(C_p)-n\right]_{r_1s_1+(r_2-1)s_2+(r_3-1)s_3}}$$

$$\times \frac{\left[1-(F_h)-n\right]_{r_1s_1+r_2s_2+r_3s_3} (\sigma_1)\left[(\alpha_u)\right]_{s_2} \left[(\alpha_m^1)\right]_{s_3}}{\left[1-(E_g)-n\right]_{r_1s_1+r_2s_2+r_3s_3} \left[(\beta_v)\right]_{s_2} \left[(\beta_k^1)\right]_{s_3}}$$

$$\times \frac{\mu_1^{s_1} x_1^{(c-d)r_1s_1} (-1)^{r_1(p+q+g+h+1)} \left(\mu_2 x_2^{r_2}\right)^{s_2}}{s_1! \quad \nu_n^{s_1} \quad \left(\mu x_1^r\right)^{r_1s_1} \quad s_2! \quad \left(\mu x_1^r\right)^{r_2s_2}}$$

$$\times \frac{(-n)_{r_1s_1+r_2s_2+r_3s_3} (-1)^{[r_2(p+q+g+h+1)+p+q]_{s_2}} \left(\mu_3 x_3^{r_3}\right)^{s_3} (-1)^{[r_3(p+q+g+h+1)+p+q]_{s_3}}}{s_3! \qquad \left(\mu x_1^r\right)^{r_1s_1}} \qquad \ldots (2.4)$$

Where

$$R = \frac{\left[(C_p)\right]_n \left[(E_g)\right]_n \left(\mu x_1^r\right)^{r_1s_1}}{\left[(D_q)\right]_n \left[(F_h)\right]_n \; n!}$$

And the single terminating factor $(-n)_{r_1s_1+r_2s_2+r_3s_3}$ makes all summation runs to ∞.

3. LAURICELLA FORM REPRESENTATION OF $B_n(x_1, x_2, x_3)$

Different forms of (2.1) have arrived at on specializing the parameters, which are given in the forms of corollaries.

Corollary I : For $r_2 > 1$ and $r_3 > 1$, we arrive at

$$B_n(x_1,x_2,x_3)=\nu_a^{-\sigma_1}F_{p+g:\,u:m}^{2+q+h:v:k}\Bigg[\big[(-n):r,r_1,r_2,r_3\big] \atop \overline{\qquad\qquad},$$

$$\left[\left(1-\left(D_q\right)-n\right):r,r_1,r_2-1,r_3-1\right],\left[\left(1-\left(F_h\right)-n\right):r,r_1,r_2,r_3\right)\right]$$
$$\left[\left(1-\left(C_p\right)-n\right):r,r_1,r_2-1,r_3-1\right],\left[\left(1-\left(E_g\right)-n\right):r,r_1,r_2,r_3\right)\right]$$

$$\left[(\sigma_1):1\right],\left[(\alpha_u):1\right],\left[\left(\alpha_m^1\right):1\right] \atop \left[(\beta_v):1\right],\left[\left(\beta_k^1\right):1\right]\overline{\qquad\quad}, \quad \frac{\mu_1 x_1^{(c-d)_{r_1}}(-1)^{r_1(p+q+g+h+1)}}{\nu_n\left(\mu x_1^r\right)^{r_1}}$$

$$\frac{\mu_2 x_2^{r_2}(-1)^{[r_2(p+q+g+h+1)+p+q]}}{\left(\mu x_1^r\right)^{r_2}},\frac{\mu_3 x_3^{e_3}(-1)^{r_3(p+q+g+h+1)+p+q}}{\left(\mu x_1^r\right)^{r_3}}\Bigg] \qquad \ldots (3.1)$$

Corollary II : For $r_2 > 1$ and $r_3 > 1$, we arrive

$$B_n(x_1,x_2,x_3)=\nu_a^{-\sigma_1}RF_{p+g:\,u:m}^{2+q+h:v:k}\Bigg[\big[(-n):r,r_1,r_2,r_3\big] \atop \overline{\qquad\qquad},$$

$$\left[\left(1-\left(D_q\right)-n\right):r,r_1,0,r_3-1\right],\left[\left(1-\left(F_h\right)-n\right):r,r_1,1,r_3\right)\right]$$
$$\left[\left(1-\left(C_p\right)-n\right):r,r_1,0,r_3-1\right],\left[\left(1-\left(E_g\right)-n\right):r,r_1,1,r_3\right)\right]$$

$$\left[(\sigma_1):1\right],\left[(\alpha_u):1\right],\left[\left(\alpha_m^1\right):1\right] \atop \left[(\beta_v):1\right],\left[\left(\beta_k^1\right):1\right]\overline{\qquad\quad}, \quad \frac{\mu_1 x_1^{(c-d)_{r_1}}(-1)^{r_1(p+q+g+h+1)}}{\nu_n\left(\mu x_1^r\right)^{r_1}}$$

$$\frac{\mu_2 x_2^{r_2}(-1)^{2(p+q)+g+h+1}}{\left(\mu x_1^r\right)},\frac{\mu_3 x_3(-1)^{r_3(p+q+g+h+1)+(p+q)}}{\left(\mu x_1^r\right)^{r_3}}\Bigg] \qquad \ldots (3.2)$$

Corollary III : For $r_2 > 1$ and $r_3 > 1$, we get

$$B_n(x_1,x_2,x_3)=\nu_a^{-\sigma_1}RF_{p+g:\,u:m}^{2+q+h:v:k}\Bigg[\big[(-n):r,r_1,r_2,r_3\big] \atop \overline{\qquad\qquad},$$

$$\left[\left(1-\left(D_q\right)-n\right):r,r_1,r_2-1,0\right],\left[\left(1-\left(F_h\right)-n\right):r,r_1,r_2,1\right)\right]$$
$$\left[\left(1-\left(C_p\right)-n\right):r,r_1,r_2-1,0\right],\left[\left(1-\left(E_g\right)-n\right):r,r_1,r_2,1\right)\right]$$

$$\left[(\sigma_1):1\right],\left[(\alpha_u):1\right],\left[(\alpha_m^1):1\right] \quad \frac{\mu_1 x_1^{(c-d)r_1}(-1)^{r_1(p+q+g+h+1)}}{\nu_n\left(\mu x_1^r\right)^{r_1}}$$

$$\left[(\beta_v):1\right],\left[(\beta_k^1):1\right]\text{———},$$

$$\left.\frac{\mu_2 x_2^{r_2}(-1)^{r_2(p+q)+g+h+1}}{\left(\mu x_1^r\right)^{r_2}},\frac{\mu_3 x_3(-1)^{2(p+q)+g+h+1}}{\left(\mu x_1^r\right)}\right] \quad \ldots (3.3)$$

Corollary IV : For $r_2 = 1 = r_3$, we obtain

$$B_n(x_1,x_2,x_3) = \nu_a^{-\sigma_1} R F_{p+g:\,u:m}^{2+q+h:v:k}\left[\begin{matrix}\left[(-n):r,r_1,1,1\right]\\ \text{———},\end{matrix}\right.$$

$$\left[\left(1-(D_q)-n\right):r,r_1,0,0\right],\left[\left(1-(F_h)-n\right):r,r_1,1,1\right]$$

$$\left[\left(1-(C_p)-n\right):r,r_1,0,0\right],\left[\left(1-(E_g)-n\right):r,r_1,1,1\right]$$

$$\left[(\sigma_1):1\right],\left[(\alpha_u):1\right],\left[(\alpha_m^1):1\right] \quad \frac{\mu_1 x_1^{(c-d)r_1}(-1)^{r_1(p+q+g+h+1)}}{\nu_n\left(\mu x_1^r\right)^{r_1}}$$

$$\left[(\beta_v):1\right],\left[(\beta_k^1):1\right]\text{———},$$

$$\left.\frac{\mu_2 x_2(-1)^{2(p+q)+g+h+1}}{\left(\mu x_1^r\right)},\frac{\mu_3 x_3(-1)^{2(p+q)+g+h+1}}{\left(\mu x_1^r\right)}\right] \quad \ldots (3.4)$$

SPECIAL CASES :

Case I : On making the substitution $s_2 = 0 = s_3 \Rightarrow x_2 = 0 = x_3$ in (2.1), we arrive at

$$B_n(x_1,0,0) = R\sum_{s_1=0}^{\infty}\frac{(-n)_{r_1 s_1}\left[1-(D_q)-n\right]_{r_1 s_1}\left[1-(F_h)-n\right]_{r_1 s_1}}{\left[1-(C_p)-n\right]_{r_1 s_1}\left[1-(E_g)-n\right]_{r_1 s_1}}$$

$$\times\frac{(\sigma_1)_{s_1} x_1^{(c-d)r_1 s_1}(-1)^{r_1(p+q+g+h+1)s_1}}{\nu_a^{s_1}\left\{\mu x_1^{(c+d)}\right\}^{r_1 s_1} \quad s_1!} \quad \ldots (3.5)$$

Case II : If we set $s_2 = 0 = s_3 \Rightarrow x_1 = 0 = x_3$ in (2.2), we get

$$B_n(0,x_2,0) = R\sum_{s_2=0}^{\infty}\frac{(-n)_{r_2 s_2}\left[1-(D_q)-n\right]_{r_2 s_2}\left[1-(F_h)-n\right]_{r_2 s_2}}{\left[1-(C_p)-n\right]_{r_2 s_2}\left[1-(E_g)-n\right]_{r_2 s_2}}$$

$$\times \frac{\left[(\alpha_u)\right]_{s_2} (\sigma_1)_{s_1} x_1^{(c-d)r_1 s_1} (-1)^{r_1(p+q+g+h+1)s_1}}{\left[(\beta_v)\right]_{s_2} \quad s_2! \quad \left(\mu x_1^r\right)^{r_2 s_2}} \qquad \ldots (3.6)$$

Case III : On setting $s_1 = 0 = s_2 \Rightarrow x_1 = 0 = x_2$ in (2.3), we achieve

$$B_n(0,0,x_3) = R\sum_{s_3=0}^{\infty} \frac{(-n)_{r_3 s_3} \left[1-(D_q)-n\right]_{r_3 s_3} \left[1-(F_h)-n\right]_{r_3 s_3}}{\left[1-(C_p)-n\right]_{r_3 s_3} \left[1-(E_g)-n\right]_{r_3 s_3}}$$

$$\times \frac{\left[(\alpha_m^1)\right]_{s_3} \mu_3^{s_3} x_3^{r_3 s_3} (-1)^{r_3(p+q+g+h+1)s_3}}{\left[(\beta_k^1)\right]_{s_3} \quad s_3! \quad \left(\mu x_1^r\right)^{r_3 s_3}} \qquad \ldots (3.7)$$

4. PARTICULAR CASES

In specializing the various para meters involved in Lauricella form, a number of interesting known and un known polynomials can be, obtained as particular cases of the generalized polynomial set $B_n(x_1, x_2, x_3)$ some of them, which are well known, are listed below :

[A] Separating the term corresponding to (3.5), we obtain a number results on specializing the remaining parameters :

(i) Srivastava K.N. Polynomial [8]

On making the substitution $p = 0 = q = h = c = d$; $g = 1 = \nu = \mu = \mu_1 = r = \sigma_1$; $E_1 = 1 + \lambda$ and $x_1 = \frac{1}{y}$ in (3.5), we have

$$B_n\left(\frac{1}{y},0,0\right) = \frac{y^{-n}(1+\lambda)_n(-1)^n}{n!} \times {}_1F_1\left[\begin{matrix}-n; \\ & -y \\ -\lambda-n;\end{matrix}\right] = y^{-n}A_n^{(\lambda)}(y) \qquad \ldots (4.1)$$

where $A_n^{(\lambda)}(y)$ are the Srivastava's Polynomials.

(ii) Pseudo Laguerre Polynomial [7]

If we take $p = 0 = q = h = c = d$; $g = 1 = \sigma_1 = \frac{1}{x} = \nu_a = \mu = \mu_1$;

$$B_n\left(\frac{1}{x},0,0,\right) = \frac{x^{-n}(-\lambda)_n}{n!} F\left[\begin{matrix}-n; \\ & x \\ 1+\lambda-n;\end{matrix}\right] = x^{-n} f_n(x) \qquad \ldots (4.2)$$

where $f_n(x)$ [7; P 245(12)] are the Pseudo Laguerre Polynomials.

(iii) Meixner Polynomials

On taking $p = 0 = q = h = r = d$; $g = c = 1 = r_1 = \mu = \mu_1 = \nu$; $x_1 = \frac{1}{C}$, $\sigma_1 = -x$, $E_1 = \beta_1 + x$ in (3.5), we achieve

$$B_n\left(\frac{1}{C}, 0, 0,\right) = \frac{(\beta_1 + x)_n}{n!} F\begin{bmatrix} -n, x; & \\ & \frac{1}{C} \\ 1 - \beta_1 - x - n; & \end{bmatrix} = m_n(x; \beta_1, c) \quad \dots (4.3)$$

where $m_n(x; \beta_1, C)$ are the Meixner Polynomials.

[B] Separating the term corresponding to (3.6), we obtain a number results on specializing the remaining parameters :

(iv) Abdul Halim and Al-Salam Polynomials [1]

If we take $p = 0 = q = g = h; = \nu$; $u = 1 = \nu = \mu_2 = \mu = x_1$; $\alpha_1 = (a_p)$, $\beta_1 = (b_q)$ and $x_2 = x$ in (3.6), we have

$$n! B_n(0, 1, 0) = F\begin{bmatrix} -n, (a_p); & \\ & x \\ (b_q); & \end{bmatrix} = {}_1F_1(-n; b; y) \quad \dots (4.4)$$

where ${}_1F_1(-n; b; x)$ are the Abdul Halim and Al-Salam Polynomials

(v) Simple Lagurre Polynomial [7]

On putting $p = 0 = q = g = h$; $r = 1 = r_2 = \mu = u = x_2 = \nu$; $\mu_1 = -1$; $\nu = 1, 2$; $\alpha_1 = 1 + \beta$, $\beta_1 = 1$, $\beta_2 = 1 + \alpha$; and writing $\frac{1}{x}$ for x_1, we have

$$B_n\left(\frac{1}{x}\right) = \frac{x^{-n}}{n!} {}_2F_2\begin{bmatrix} -n, 1+\beta; & \\ & x \\ 1, 1+\alpha; & \end{bmatrix} = \frac{x^{-n}}{n!} \phi_n(x) \quad \dots (4.5)$$

where $\phi_n(x)$ [7; P 235(12)] reduces to simple Laguerre Polynomials for $\alpha = \beta$.

(vi) The Polynomial's $f_n(x)$ [7; P-243(4)]

On putting $p = 0 = q = g = h = u$; $r = 1 = r_2 = \mu = \mu_1 = \nu = \nu$; $\nu = 1$ or 2, $\beta_1 = 1 + \alpha$, $\beta_2 = 1 + \beta$ and putting $\frac{1}{x}$ for x_1, we have

$$B_n\left(\frac{1}{x}\right) = \frac{x^{-n}}{n!} {}_1F_2\begin{bmatrix} -n; & \\ & x \\ 1+\alpha; 1+\beta; & \end{bmatrix} = \frac{x^{-n}}{n!} f_n(x) \quad \dots (4.6)$$

where $f_n(x)$ are the polynomials defined in [7].

(vii) Lommel Polynomial's

On putting $p = q = 0 = h = u$; $r = 1 = \nu = x_2 = \nu$; $g = 1$ or 2, $E_1 = 1$; $E_2 = \nu = \beta_1$; $r_2 = 2 = \mu = \mu_2 = 4$, and z for x_1 in (3.6) we get

$$B_n(z,1,0) = \frac{(\nu)_n (2z)^n}{n!} F\left[\begin{matrix} \frac{-n}{2}, \frac{-n}{2}+\frac{1}{2}; \\ \qquad \frac{-1}{z^2} \\ 1-\nu, -n, 1-n-\nu; \end{matrix}\right] = R_n \nu\left(\frac{1}{z}\right) \qquad \ldots (4.7)$$

where $R_n \nu\left(\frac{1}{z}\right)$ are the Lommel Polynomials [7; P. 112(5)].

[C] Separating the term corresponding to (3.7), we obtain a number results on specializing the remaining parameters :

(viii) Gould-Hopper Polynomials [5]

Putting the value $p = 0 = q = g = h; = m = k$; $r_3 = m = x_3 = r = 1 = \nu = \mu = \mu_3 = h$, and $x_1 = x$ in (3.7), we set

$$B_n(x,0,m) = \frac{x^n}{n!}\; {}_mF_0\left[\begin{matrix} \Delta(m;-n), \\ \qquad h\left(\frac{-m}{x}\right)^m \\ \text{—}; \end{matrix}\right] = \frac{1}{n!} g_n^m(x,h) \qquad \ldots (4.8)$$

where $g_n^m(x,h)$ are the Gould-Hopper Polynomials.

(ix) Bragg Polynomial [3]

On putting $p = 0 = q = g = h = m = k$; $r = 1 = \nu = \mu$; $\mu_3 = -1$; $r_3 = p = x_3$; py for x_1 in (3.7), we get

$$B_n(py,0,p) = \frac{(py)^n}{n!}\; {}_pF_0\left[\begin{matrix} \Delta(p;-n); \\ \qquad -\left(\frac{-1}{x}\right)^p \\ \text{—}; \end{matrix}\right] = \frac{1}{n!}\, g_n^p(y) \qquad \ldots (4.9)$$

where $g_n^p(y)$ are the Bragg Polynomials.

(x) Lahiri Polynomials[6]

On taking $p = 0 = q = g = h = m = k$; $r = 1 = \nu = \nu = \mu_3$; $x_3 = m = r_3$; $\mu = \nu$, $x_1 = x$ in (3.7), we get

$$B_n(x,0,m) = \frac{(\nu x)^n}{n!}\ {}_mF_0\left[\begin{matrix}\Delta(m;-n); \\ \qquad -\left(\frac{-m}{\nu x}\right)^m \\ —;\end{matrix}\right] = \frac{1}{n!} H_{n,m,\nu}(x) \qquad \ldots (4.10)$$

where $H_{n,m,\nu}(x)$ are the generalized Polynomials defined by Maya Lahiri [6].

(xi) Brafman Polynomials[2]

On putting $p = 0 = q = g = h$; $m = 1 = k = x = \mu = r = x_3 = a = 1$; σ_1; $\mu_3 = x$; $\nu = (-p)^{-n}$ and $(\alpha_1{}^1) = (\alpha_u)$, $(\beta_1{}^1) = (\beta_\nu)$ in (3.7) we have

$$B_n(1,0,1) = \frac{(-p)^n}{n!}\ {}_{1+u}F_\nu\left[\begin{matrix}\Delta(p;-n),\ (\alpha_u); \\ \qquad x \\ (\beta_\nu);\end{matrix}\right] = \beta_n^p(x) \qquad \ldots (4.11)$$

where $\beta_n^p(x)$ is the generalization of the Hermite Polynomials by Brafman [2].

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